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The behaviour analysis of an anaerobic sludge digester exposed on thermic action*

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Abstract: The anaerobic sludge digesters existing in wastewater treatment plants are exposed to temperature variations, due to permanent sludge heating at 35°C - 37°C and external weather temperature variation. This work presents the behaviour in time, as well as the calculation analysis of thermic effects action diminution for that kind of things, taking into account a small energetic balance.

Keywords: rigidity, thermoelasticity, thermal action.

List of variables and constants

E - elasticity module;
μ - Poisson coefficient;
α_{t}- - coefficient of thermal dilatation;
α_i, α_e - coefficient of thermal transfer at the exterior surface, respectively at the interior surface of the tank;
λ_b, λ_b - coefficient of thermal conduction of concrete, respectively of thermal insulation;
h - thickness of the plate;
$D = \dfrac{E \cdot h}{1-\mu^2}$ - stretching stiffness of the plate in elastic state;
$B = \dfrac{E \cdot h^3}{12(1-\mu^2)}$ - bending stiffness of the plate in elastic state;
R_i, R_s - the rays of the inferior contour, respectively the superior one of the tronconical plate;
$T_i(r)$, $T_i(x)$ -temperature function on the interior face of the plate;
$T_e(r)$, $T_e(x)$ -temperature function on the exterior face of the plate;
$T_0(r)$, $T_0(x)$ - the uniform component of temperature field upon the thickness of the plate;
$\Delta T_0(r)$, $\Delta T_0(x)$ - the linear component of temperature field upon the thickness of the plate;
X(r), X(x) - the component of the external loads on radius direction, respectively on the generating line direction;
Z(r), Z(x) - the component of the external loads on plate surface;

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α - the angle between the generating line of the tronconical plate and the horizontal plane;
k- the amortization factor of the cylindrical plate;
a - the medium ray of the cylindrical plate;
u(r), u(x)- the plate deformation on the ray direction, respectively on the generating line direction of the plate;
w(r), w(x)- the plate deformation on normal direction on the surface of the plate;
$\Delta_r(x)$ - the curved plate deformation on radius direction;
$\chi_r(r)$, $\chi_x(x)$ - plate rotation on radius direction, respectively on the generating line direction;
$N_r(r)$, N_θ (r), $N_x(x)$ - axial force on circular and curved plates;
$M_r(r)$,M_θ (r), $M_x(x)$ - bending moment on circular and curved plates;
$Q_r(r)$, $Q_x(x)$ - shearing force on circular and curved plates.

1. Introduction

The water treatment and purifing technique made necessary the accomplishment of numberless structures with different forms: cylindrical, tronconical, spherical, rectangular or combinations between them, for example cylindrical-tronconical or ovoidal form, as in case of the anaerobic sludge digester tanks.

In practice, the achievement of hydroedilitary constructions in Romania's last 100 years showed a generally corresponding behaviour of the structures, registering quite frequently the necessity of rehabilitation of the structures from the structural point of vue, resulting from damages due to lapse of time, which jeopardise the tightness and durability.
The damage causes are multiple, but one of the most important, that produce fissuration and deterioration in time of the hydroedilitary constructions structures, is the temperature variations action, determined both by seasonal variation of the exterior air temperature, the accumulated fluids temperature variation, as well as a series of other factors like: the degree of thermoisolation and the quality of the thermoinsulating materials, the presence of underground waters. For that we can name numberless deteriorations due to temperature varation as:
- fissuration above the admisible limits of circular foundation plates, immediately after placing the concrete, due to the thermal gradient produced by hidratation heat, on unprotected and untreated structures;
- fissuration and cracking of the tronconical reinforced concrete domes on anaerobic sludge digester tanks, where the thermal action is significative, because the sludge stored is heated permanently at +35 ^oC, while the exterior temperatures can vary from -25 to +40 ^oC.

In tehnical literature, the establishment of the state of efforts from the action of temperature variation, for different structural elements, was made in the hypothesis of the ideal-elastic working stage of reinforced concrete, not taking into account the damages of the rigidities in time, which provoke implicitely modifications of the state of efforts, especially due to thermal action, taking into account that the efforts produced by the thermal sollicitation depend on rigidity.

So in this work we present an analysis of the effect produced in time by the temperature variation, in the structure of an anaerobic sludge digester tank, with a stiffness degradation coefficient variable in time.

2. Determination of the effort state produced in the tank's structure from the simultaneously action of charges and temperature variations.

We shall study the case of anaerobic sludge digesters tanks from reinforced concrete (C25/30), with a capacity of 4000 m^3, with a classical cylindro-tronconical form, presented in figure 1 with the following characteristics:

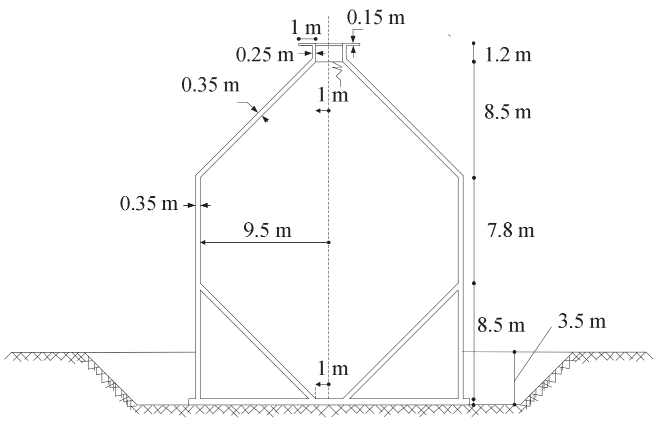

Fig. 1 The anaerobic sludge digester tank. Vertical section.

2.1. Calculus hypothesis

This state of efforts and deformations analysis has the following basic hypothesis:
- simplifying hypothesis from the linear-elastic theory for plane and curved bending plates, stated by Kirchhoff [1],[2];
- linear temperature variation on the thickness of the plate;
- temperature functions on the interior face of the plate T_i and the exterior face T_e are defined by thermal transfer calculations, in a stationary regime.

In order to obtain calculations facilities, the thermal field which take action upon the plate may be split into two components (figure 2) as follows:
- a uniform component on the thickness of the plate, T_0 which lead to the development of sectional axial efforts;

- a linear component on the thickness of the plate T_0 , with zero value in the median surface, which produce sectional bending efforts.

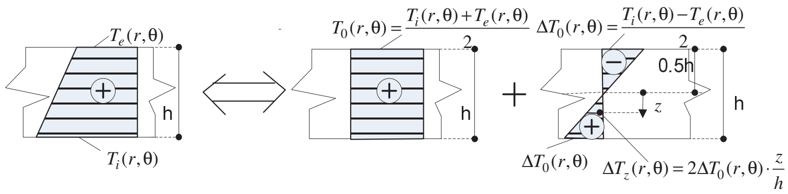

Fig. 2 The decomposing of the thermic field in elementary components when the circular plane plate is concerned

In order to establish the state of efforts produced by the permanent charges and by the temperature variation, we shall use the general efforts method. We must state that due to the fact that the efforts resulted from the permanent or cvasi-permanent actions do not depend on the stiffness, we shall consider that the structure works in the elastic stage. In same time, we shall consider that, as is described in technical literature, it is produced a degradation of stiffness and a variation in time of values of efforts state produced by thermal action.

From the analysis of numberless damages produced at the anaerobic sludge digester tanks made in Romania it has been noticed that the zone in which the most important structural damages appear is that of the tronconical roof, where due to the combined action of the hydrostatic pressure and temperature variation, the concrete might be fissurated, leading to a decrease of the tightness and implicitly of the durability. So, we can study only the structural ensemble tronconical roof - cylindrical wall.

If we take into account the physical, geometrical schematism and the links, we have the real structure calculus model. The basic system is obtained by suppressing the continuity links between the structural elements.

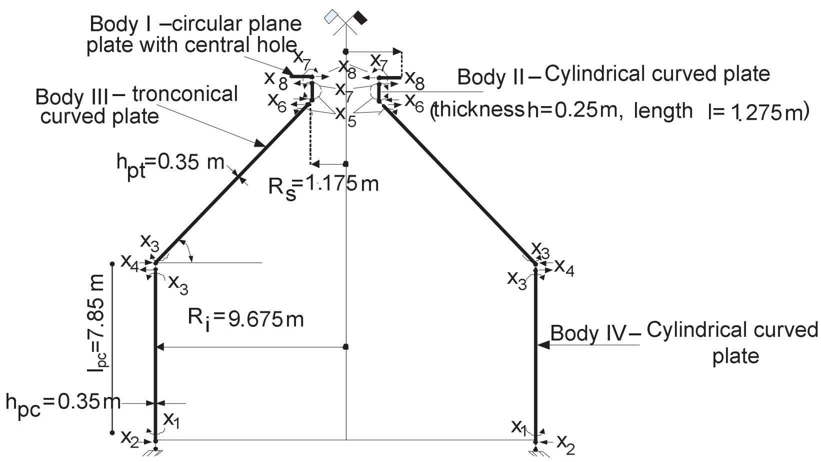

Fig. 3 Basic system of the structural part cylindrical wall - tank roof

2.2. Establishment of the efforts state produced in body 1 (circular plane plate)

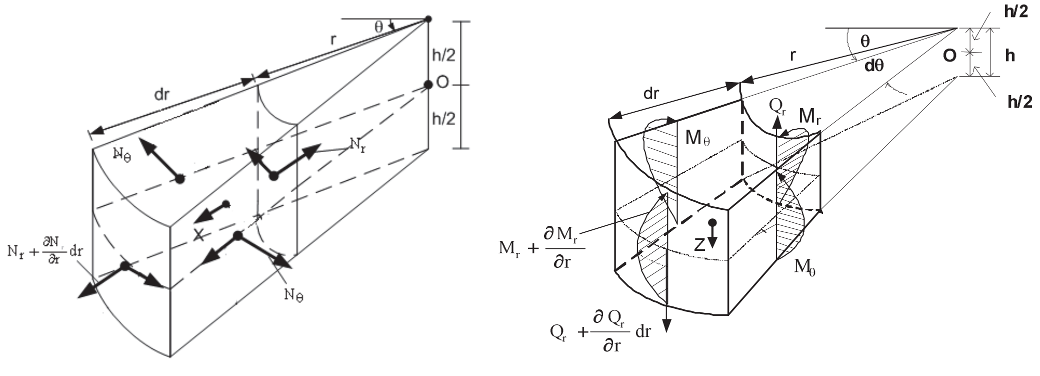

Fig. 4 The sectional pozitive efforts acting on an infinitesimal plate element

The plate deformations and sectional efforts are calculated with the relations:

$$u(r) = u_p(r) + C_1 \cdot r + C_2 \cdot r^{-1} \tag{1}$$

$$w(\rho) = w_p(\rho) + C_3 + C_4\rho^2 + C_5\rho^2 \ln\rho + C_6 \ln\rho \tag{2}$$

$$\chi_r(r) = -\frac{dw(r)}{dr} \tag{3}$$

$$N_r(r) = D \cdot \left[\frac{du(r)}{dr} + \mu \frac{u(r)}{r} - (1+\mu)\alpha_t T_0(r) \right] \tag{4}$$

$$N_\theta(r) = D \cdot \left[\frac{u(r)}{r} + \mu \frac{du(r)}{dr} - (1+\mu)\alpha_t T_0(r) \right] \tag{5}$$

$$M_r(r) = -B \left[\frac{d^2w(r)}{dr^2} + \frac{\mu}{r} \cdot \frac{dw(r)}{dr} + 2(1+\mu) \cdot \frac{\alpha_t \Delta T_0(r)}{h} \right] \tag{6}$$

$$M_\theta(r) = -B \left[\frac{1}{r} \cdot \frac{dw(r)}{dr} + \mu \cdot \frac{d^2w(r)}{dr^2} + 2(1+\mu) \cdot \frac{\alpha_t \Delta T_0(r)}{h} \right] \tag{7}$$

while the $C_1..C_6$ are integration constants and u_p, w_p are the particular solutions of the synthesis differential equations which define the state of efforts and deformations [1],[2].

2.3 Establishment of the efforts state produced in bodies 2 and 4 (cylindrical curved plates)

The synthesis equation which define the state of axial-symetrical efforts and deformations that appear in the cylindrical curved plate by the simultaneously action of the charges and temperature variations is the following [1],[3]:

$$\frac{d^4w}{dx^4}+4k^4\cdot w=-\frac{Z(x)}{B}+4k^4\cdot a\cdot\alpha_t\cdot T_0(x)+2\cdot(1+\mu)\cdot\frac{\alpha_t}{h}\cdot\frac{d^2\Delta T_0(x)}{dx^2} \tag{8}$$

The deformation plate and sectional efforts have the expressions (9),..(14):

$$w(x)=w_p(x)+C_7\,ch(kx)\cos(kx)+C_8\,ch(kx)\sin(kx)+C_9 sh(kx)\cos(kx)+C_{10}sh(kx)\sin(kx)$$

$$\chi_x=-\frac{dw(x)}{dx}$$

$$N_x(x)=-\int X(x)dx+C_{11} \tag{11}$$

$$N_\theta(x)=D\left[\frac{w(x)}{a}-(1+\mu)\alpha_t T_0(x)\right] \tag{12}$$

$$M_x(x)=B\left[\frac{\partial^2 w(x)}{\partial x^2}-2(1+\mu)\frac{\alpha_t}{h}\Delta T_0(x)\right] \tag{13}$$

$$M_\theta(x)=B\left[\mu\frac{\partial^2 w(x)}{\partial x^2}-2(1+\mu)\frac{\alpha_t}{h}\Delta T_0(x)\right] \tag{14}$$

where $w_p(x)$ is the particular solution of the differential synthesis equation and the $C_7..C_{11}$ are the integration constants.

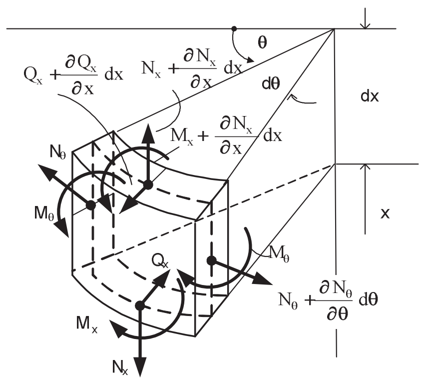

Fig. 5 Sectional positive efforts acting on an infinitesimal element of the cylindrical plate

2.4 Establishment of the state of efforts produced in body 3 (tronconical curved plate)

The synthesis equation which define the axial-symmetrical state of efforts and deformations produced in the tronconical curved plate by the simultaneously action of external loads and temperature variations is:

$$L(L(Q_x x) + \frac{E \cdot h \cdot tg(\alpha)}{B} \cdot Q_x \cdot x = \frac{1}{B} \cdot \left[\frac{dZ(x)}{dx} + \frac{2Z(x)}{x} - \frac{1}{x^3} \left(\int Z(x) + X(x) ctg(\alpha) \right) x dx - \right.$$
$$\left. - \frac{\mu X(x) tg(\alpha)}{x} \right] - E \cdot h \cdot tg(\alpha) \left[\alpha_t L \left(\frac{d T_0(x)}{d x} \cdot x \right) + 2(1+\mu)\alpha_t \frac{d \Delta T_0(x)}{d x} \cdot x \cdot tg(\alpha) \right] \tag{15}$$

$$L() = \frac{d^2}{d x^2}() + \frac{d}{d x}() - \frac{1}{x} \cdot () . \tag{16}$$

If we define $\lambda = \sqrt[4]{\frac{12 \cdot (1-\mu^2) \sin^2(\alpha)}{(R_i + R_s) \cdot h^2}}$ the solution of the equation (15) has the form [1],[2]:

$$Q_x(x) = Q_p(x) + C_{12}\, ch(\lambda x) \cos(\lambda x) + C_{13}\, ch(\lambda x) \sin(\lambda x) +$$
$$+ C_{14} sh(\lambda x) \cos(\lambda x) + C_{15} sh(\lambda x) \sin(\lambda x) - 2B(1+\mu) \frac{\alpha_t}{h} \frac{d\Delta T_0(x)}{dx} \tag{17}$$

where the C_{12}..C_{16} the integrations constants. The axial-symmetrical state of efforts produced in the tronconical curved plate by the simultaneously action of charges and temperature variations are calculated with the relations:

$$N_x(x) = Q_x(x) ctg(\alpha) - \frac{1}{x} \int \left(X(x) + Z(x) ctg(\alpha) \right) x dx + C_{16} \tag{18}$$

$$N_\theta(x) = \frac{R_i + R_s}{2 \sin(\alpha)} \cdot \frac{d Q_x(x)}{d x} - Z(x) x ctg(\alpha) \tag{19}$$

$$M_x(x) = -B \frac{d\chi_x(x)}{d x} - 2B(1+\mu)\alpha_t \frac{\Delta T_0(x)}{h} \tag{20}$$

$$M_\theta(x) = -\mu B \frac{d\chi_x(x)}{d x} - 2B(1+\mu)\alpha_t \frac{\Delta T_0(x)}{h} \tag{21}$$

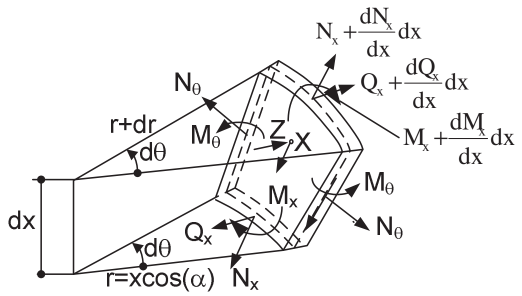

Fig. 6. Sectional positive efforts acting on an infinitesimal element of the tronconical curved plate

Deformation of the plate χ_x and Δ_r may be calculated with the relations:

$$\chi_x(x) = -\frac{\cot(\alpha)}{E\cdot h}\left[\left(N_\theta - N_x\right)\cdot(1+\mu) + x\cdot\left(\frac{dN_\theta}{dx} - \mu\cdot\frac{dN_x}{dx}\right)\right] \tag{22}$$

$$\Delta_r(x) = \frac{x\cdot\cos(\alpha)}{E\cdot h}\cdot\left(N_\theta - \mu N_x\right) + x\cdot\cos(\alpha)\cdot\alpha_t\cdot T_0(x) \tag{23}$$

3. Results

In order to determine the efforts state produced by the cumulated action of the charges and of the thermic solicitation we shall write the compatibility equations system (24) between the real system and the basic system. If we define the matrix $\Delta_i=\Delta_{i,p}$, then the system (24) may be written in a matric form (i,j=1..8):

$$\delta_{ij}x_j + \Delta_i = 0 \tag{24}$$

where Δ_{ij} is the flexibility matrix of the studied structure, and $\Delta_{i,p}$ are the displacements produced by the external loads action on the basical system. The unitary displacements and those produced by the external loads and temperature variations on the basic system are calculated by the relations (1), (2), (3), (11), (12), (21), (22). The state of efforts is obtained by supperposing the state of efforts produced by the unknown variables on the basic system and the state of effort produced by the external loads and the temperature variation (fig.7), taking into account the relations (6)-(10),(15)-(19),(22)-(29).

In figure 8 and table 1 we present the state of efforts in the tronconic roof – cylindric wall part, considering the elastic stiffness hypothesis (hypothesis 1), the hypothesis of the stiffness which is diminuated by 25% compared to the elastic stiffness (hypothesis 2), respectively the hypothesis of the stiffness diminuated by 50% (hypothesis 3), both in case of a noninsulated structure and in case of the thermic insulated structure with a 10 cm mineral cotton layer (α_i = 500 W/(m^{2o}C), α_e = 24 W/(m^{2o}C, α_b=1.74 W/(m^{2o}C), α_{iz}=0.04 W/(m^{2o}C), α_t=10^{-5} ^{o}C^{-1}).

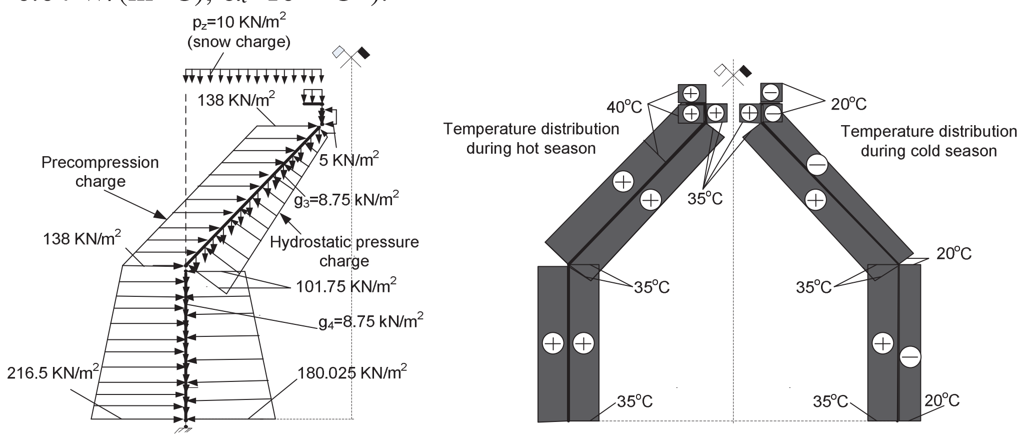

Fig 7 Exterior charges and temperature distribution inside, respectively outside of the tank considered for the studied case

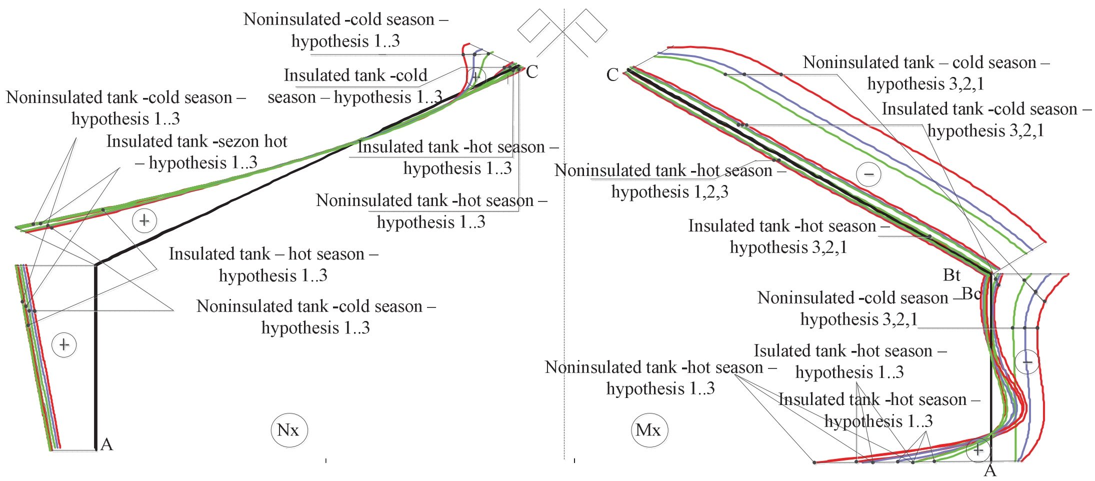

Fig 8 Sectional effort variation N_x (N_r) and M_x in tronconical roof – cylindrical wall structural part

Table 1

Values of efforts in in tronconical roof – cylindrical wall structural part

Structural element	Section	Noninsulated tank - cold season				Isulated tank - cold season			
		Hypothesis 1		Hypothesis 3		Hypothesis 1		Hypothesis 3	
		N_x [KN]	M_x [KNm]	N_x [KN]	M_x [KNm]	N_x [KN]	M_x [KNm]	N_x [KN]	M_x [KNm]
Cylindrical plate	A	69.53	-71.09	80.67	-57.51	88.130	308.34	89.97	132.20
	Bc	138.21	-183.19	149.36	-94.15	156.82	-29.24	158.66	-17.17
Tronconical plate	Bt	195.47	-183.19	211.22	-94.15	221.77	-29.24	224.38	-17.17
	C	138.72	-133.65	38.04	-66.57	17.92	-19.01	7.63	-9.26

Structural element	Section	Noninsulated tank - hot season				Isulated tank - hot season			
		Hypothesis 1		Hypothesis 3		Hypothesis 1		Hypothesis 3	
		N_x [KN]	M_x [KNm]	N_x [KN]	M_x [KNm]	N_x [KN]	M_x [KNm]	N_x [KN]	M_x [KNm]
Cylindrical plate	A	93.28	413.29	92.54	184.68	91.69	381.03	91.75	168.53
	Bc	161.96	13.34	161.23	4.12	160.38	0.24	160.44	-2.43
Tronconical plate	Bt	239.05	13.34	228.01	4.12	236.81	0.24	226.89	-2.43
	C	-15.50	12.7	-9.07	6.6	-4.17	2.95	-3.41	1.72

4. Conclusions

If an analysis in time is done for the cylindrical wall- tronconical roof part, we notice that:

1. In the studied case we obtain in cold season for noninsulated tank in section C, the value of axial force N_x reduced by 72%, respectively in case of a bending moment with 50.2%, in exploitation of the tank. So we can see a variation in time of the state of efforts in the anaerobic sludge digester tank structure, that means a maximum value after the concrete hardening, because in this phase concrete works in its elastic stage, case in which we have maximum values of efforts produced by thermic action.
2. Maximum value of sectional and unitary efforts, while concrete is hardening, leads, when a insufficient reinforcement percentage lacks, to a diminution in time of the stiffness to stretching and bending. So, during exploitation, after a period of time the stiffness value diminishes slowly as soon as fissures appear and this process stabilizes. In the same time, in case of a hydroedilitary construction, to which the fissure opening is limited, there must be imposed percentages of reinforcement corelated to the maximum fissure openings accepted. Thus it appears as necessary the establishment in the future of the bending and stretching stiffness for the plane and curved plates of reinforced fissured concrete, for a better optimization of the resistance structure for hydroedilitary constructions and particularly the anaerobic sludge digesters tanks.
3. In the same time the structure optimization supposes the corelation of the reinforcement percentage to the thickness of the walls and to the degree of thermoinsulation adopted for the walls, so that the energetic efficiency of the anaerobic sludge digester tanks is maximal. For exemplification we present the graphic in figure 9 where it is shown the reinforcement percentage variation on the vertical direction, disposed in the tronconical roof (insulated tank, cold season, hypothesis 3) on the inferior contour, as a function of thermal insulated degree [3], for different reduction values of elastic rigidity.

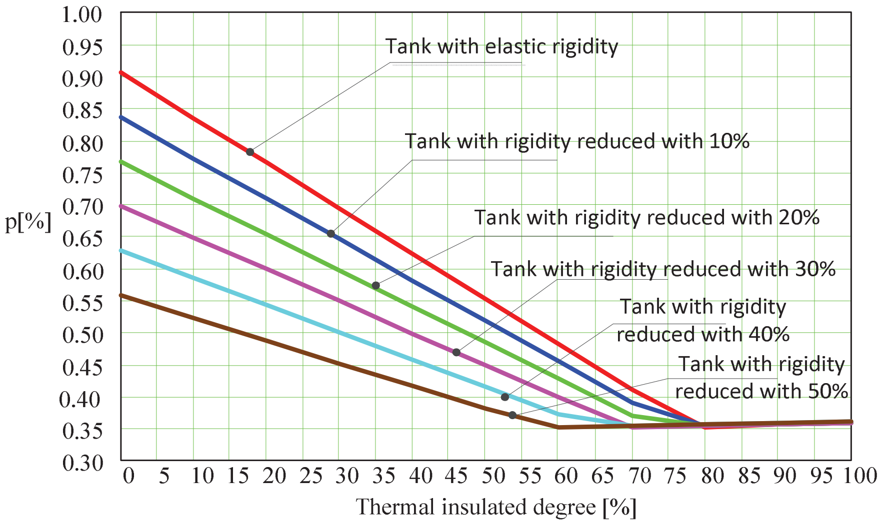

Fig. 9 Variation of the reinforcement percentage p depending upon the value of thermal insulated degree, in case of a tank made of reinforced concrete (C25/30, with reinforcing bars type PC52)

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