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The practice of spatial variability modeling in geotechnical engineering

Practica modelării variabilității spațiale în ingineria geotehnică

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Abstract. *From a set of georeferenced data (measurements and locations at which these measurements are taken), not necessarily distributed regularly, we want to find a rule to estimate the values taken by the observed parameter at other points of space different from those at which the measurements are taken. We then speak of spatial interpolation.*
Spatial interpolation, that is a process of using a set of point data to create surface data, can either be a deterministic or a stochastic (also known as geostatistical) interpolation techniques.
In the present study, two interpolation methods such as inverse distance weighting (IDW) and ordinary kriging (OK) are used to generate spatial distribution of geotechnical parameters.

Key words: *Spatial variability, spatial interpolation, inverse distance method (IDW), ordinary kriging (OK).*

1. Introduction

The problem of obtaining values, which are unknown, has drawn attention of many science researchers. For reasons of economy, there will always be only a limited number of sample points, where observations are measured.
Spatial interpolation is the process of using points with known values to estimate unknown values at other points. The control points are points with known values. They provide the data necessary for the development of an interpolator for spatial interpolation.

There are many methods of spatial interpolation. They are divided into two categories, namely: deterministic and geostatistical methods [1]-[2].

Estimation formulas for almost all spatial interpolation methods can be represented as weighted averages of sampled data [3].

There are several techniques or methods, including inverse distance writhing (IDW) and ordinary kriging (OK).

The IDW method is similar to ordinary Kriging in that it gives more weight to values close to a point, but has a lower computational complexity. IDW uses a simple distance-based algorithm (Johnston et al. 2001).

Both models, ordinary Kriging and IDW, assume that the predictions are a linear combination of the data, as the equation (1) shows (Gotway et al. 1996, Schloeder et al. 2001).

$$Z^*(S_0) = \sum_{i=1}^{n} \lambda_i Z(S_i), \ \ i = 1, \dots, n \tag{1}$$

where $Z(S_0)$ is the estimated value of an attribute at the point of interest S_0, Z(Si) is the observed value at the sampled point Si, λ_i is the weight assigned to the sampled points, and n represents the number of sampled points used for the estimation.

2. Theoretical basics

The Spatial interpolation is the process of using points with known values to estimate unknown values at other points. All interpolation methods have been developed based on this theory that closer points to each other have more correlations and similarities than farther points.

In the following, we will present two widely used spatial interpolation methods called Inverse Distance Weighting (IDW) and ordinary kriging (OK).

2.1 The inverse distance weighting

The IDW method uses the inverse function of distance to calculate the weights for each of the cell values, this method is based on the premise that the things that are closer are more similar and therefore they have a greater weight and influence on the point to calculate.

The estimate is calculated as the weighted average of the values recorded in the nearby stations, to which a weight is assigned.

The used relationships are:

$$\lambda_i = \frac{[h(S_i, S_0)]^{-p}}{\sum_{i=1}^{n} [h(S_i, S_0)]^{-p}} \ , i = 1, \dots, n \tag{2}$$

$$Z^*(S_0) = \frac{\sum_{i=1}^{n}[h(S_i,S_0)]^{-p}Z(S_i)}{\sum_{i=1}^{n}[h(S_i,S_0)]^{-p}}, i = 1, \dots, n \quad (3)$$

Where $h(S_i,S_0)$ is the the distance between S_0 and S_i, p is a power parameter, and n represents the number sampled points used for the estimation. The most popular choice is $p = 2$.

2.2 Ordinary kriging

Unlike IDW, kriging is method based on spatial autocorrelation. It uses semivariogram. Kriging is a powerful type of spatial interpolation that uses complex mathematical formulas to estimate values at unknown points based on the values at known points. Kriging measures distances between all possible pairs of sample points and uses this information to model the spatial autocorrelation for the particular surface we are interpolating. There is a large range of choices among krigings and, obviously, they produce different maps [1]-[2]-[3], [6].

The kriging estimates the unknown value using a weighted linear combinations of the available sample (1).

These weights are calculated based on the distance between the sampled points and the point where the corresponding prediction will be made.

The ordinary Kriging method obtains the weights (or influence) of the values, solving the Kriging equation shown in the equation

$$\sum_{i-1}^{n} \lambda_i \gamma[h(S_i,S_j)] + m = \gamma[h(S_i,S_0)], i = 1, \dots, n \quad (4)$$

With

$$\sum_{i=1}^{n} \lambda_i = 1 \quad (5)$$

The dependence (The spatial structure) of observations in space is approached through the space correlation functions that are the semivariogram calculation [7]-[8], [9].

$$\gamma[h(S_i,S_j)] = \frac{1}{2} var[Z(S_i) - Z(S_j)] \quad (6)$$

where *var* is the variance and

$$h(S_i,S_j) = \|S_i - S_j\| = h_{ij} \quad (7)$$

h_{ij} is the distance between S_i and S_j

For a given value of h, we obtain an unbiased estimator of $\gamma(h)$ in the following way:

$$\gamma(h) = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} [Z(S_i) - Z(S_j)]^2 \tag{8}$$

The semivariogram is a tool that allows to analyze the spatial behavior of a variable over a defined area, obtaining as result an experimental variogram that reflects the maximum distance and the way in which a point has influence on another point at different distances.

Once the experimental variogram has been calculated, a mathematical model must be made that models the experimental variogram in the best possible way, which is known as a model (theoretical) semivariogram [7]-[8].

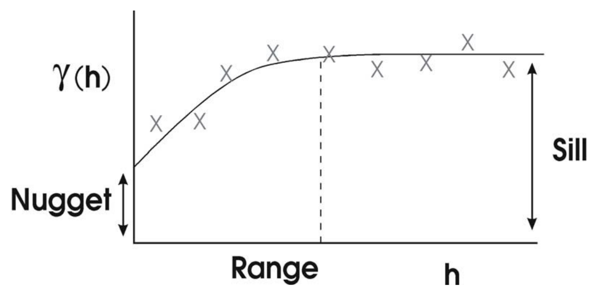

Fig. 1. Adjustment of a theoretical semivariogram (curve) to an experimental semivariogram (X).

where:
a = range, C_0 =nugget, $C_0 + C_1$ = sill, h = offset.
Several types of semivariogram models exist. For example, the exponential model is

$$\begin{cases} \gamma(h) = C_0 + C_1\left(1 - \exp\left(-\frac{h}{a}\right)\right) \\ \gamma(0) = 0 \end{cases} \tag{9}$$

and the Gaussian model is

$$\begin{cases} \gamma(h) = C_0 + C_1\left(1 - \exp(-\frac{h^2}{a^2})\right) \\ \gamma(0) = 0 \end{cases} \tag{10}$$

The ordinary Kriging method obtains the weights (or influence) of the values, solving the Kriging equation shown in the equation

$$\begin{pmatrix} \gamma(h_{11}) & \gamma(h_{12}) & \dots & \gamma(h_{1n}) & 1 \\ \gamma(h_{21}) & \gamma(h_{22}) & \dots & \gamma(h_{2n}) & 1 \\ . & . & \dots & . & . \\ . & . & \dots & . & . \\ . & . & \dots & . & . \\ \gamma(h_{n1}) & \gamma(h_{n2}) & \dots & \gamma(h_{nn}) & 1 \\ 1 & 1 & \dots & 1 & 0 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ . \\ . \\ . \\ \lambda_n \\ m \end{pmatrix} = \begin{pmatrix} \gamma(h_{10}) \\ \gamma(h_{20}) \\ . \\ . \\ . \\ \gamma(h_{n0}) \\ 1 \end{pmatrix} \quad (11)$$

where n is the number of observations, m is the Lagrange multiplier used for the minimization of constraints, λ is the weight given to each of the observations and the sum of all λ is equal to one. The subscripts i and j denote the points sampled, the subscript 0 is the point in estimation and h_{ij} is the distance between S_i and S_0 from the semivariogram.

We write (1) in a simplified way,

$$\Gamma * W = G \quad (12)$$

Multiplying by the inverse of Γ, we obtain

$$W = \Gamma^{-1} * G \quad (13)$$

Which makes it possible to obtain W that is λ_1, λ_2, ..., λ_n. The calculated coefficients will be used in formula (1) to calculate $Z^*(S_0)$.

3. Applying the inverse distance weighting and kriging methods

In this part will be presented the results of spatial variability estimation of geotechnical properties on a site.
During the field exploration phase, in situ engineering field tests are carried out. These include water content in soil (W%), the apparent density and Liquid Limit (W.L%).
The methods used for this estimation are:
Inverse distance weighting (IDW) and ordinary kriging (OK).

3.1 Study area

- Situation

The study area is a site for a gas processing center located on the east side of the national highway number 6 (R.N.6). It is located in the region of Touat (Adrar department), not far from the village of Sali.

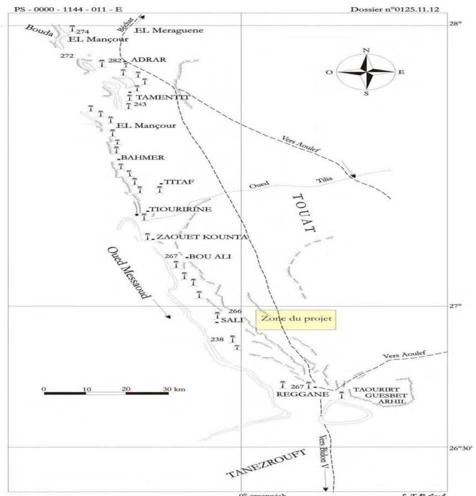

Fig. 2. Situation of the study area and access road.

The coordinates of the project site boundaries are:
C.1 (X : 210 796,000 – Y : 2 978 406,000 – Z : 268,684)
C.2 (X : 211 599,000 – Y : 2 978 406,000 – Z : 267,444)
C.3 (X : 211 599,000 – Y : 2 979 243,000 – Z : 269,394)
C.4 (X : 210 796,000 – Y : 2 979 243,000 – Z : 267,143)

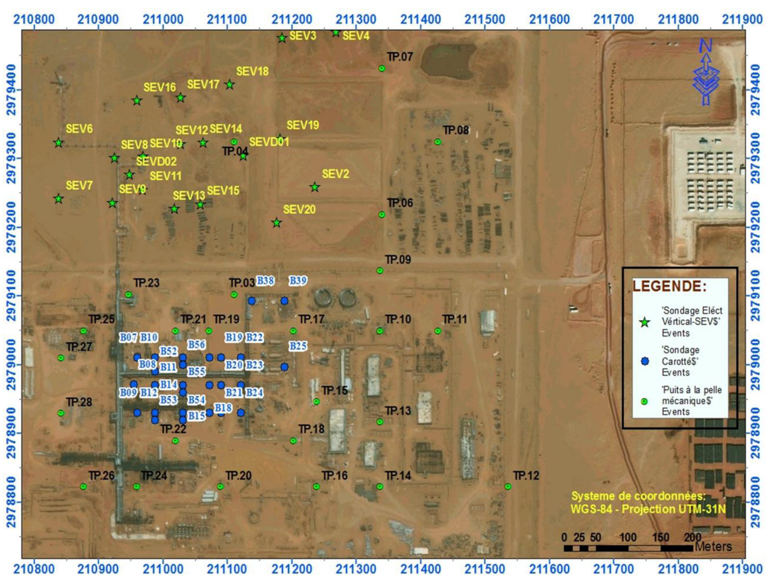

Fig. 3. The study area is a flat ground.

3.2 Results and discussions

Results

A. Spatial variability of Water content (w%)

- *With inverse distance weighting (IDW)*

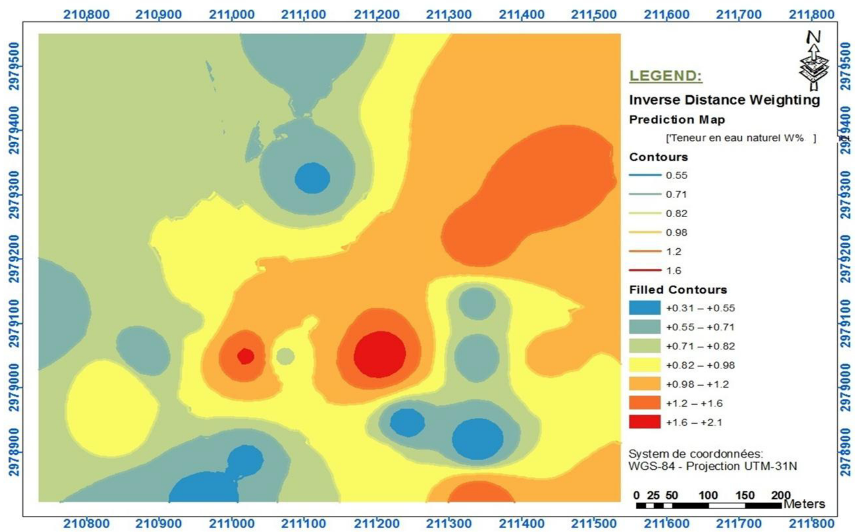

Fig. 4. Water content model obtained on the basis of the IDW spatial interpolation.

- *With ordinary kriging (OK)*

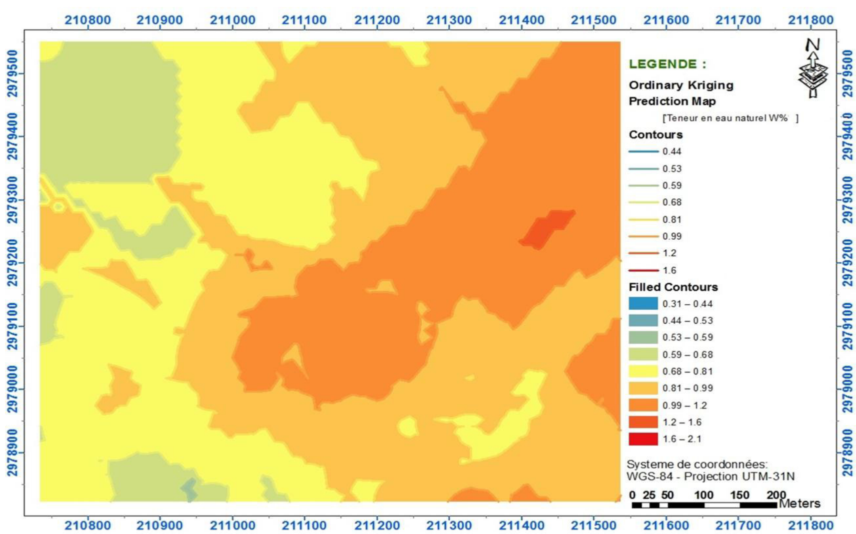

Fig. 5. Water content model obtained by the ordinary kriging method (OK).

B. Spatial variability of the apparent density

➢ *With inverse distance weighting (IDW)*

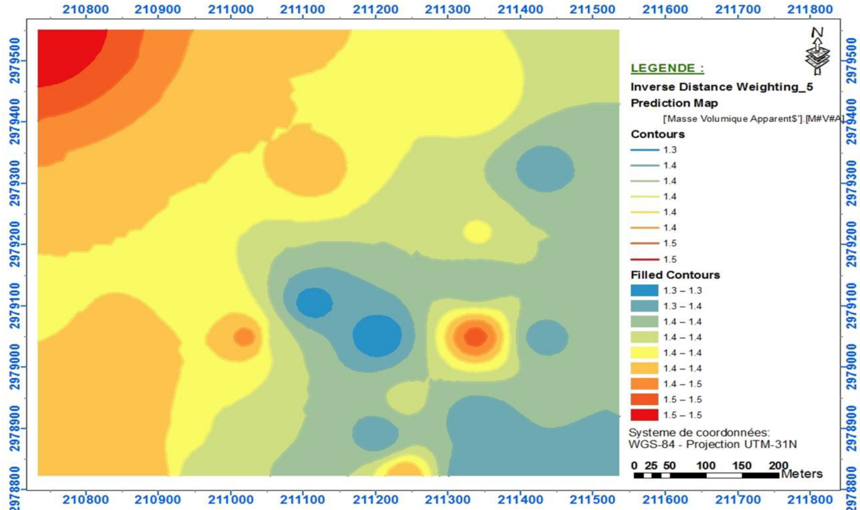

Fig. 6. Apparent density model obtained on the basis of the IDW spatial interpolation.

➢ *With ordinary kriging (OK)*

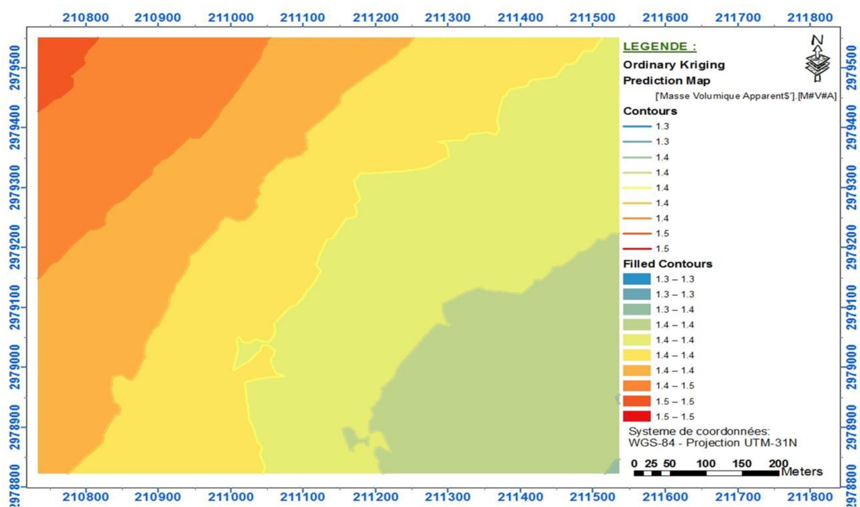

Fig. 7. Apparent density model obtained by the ordinary kriging method (OK).

C. Spatial variability of the liquid limit (W.L%)

➢ *With inverse distance weighting (IDW)*

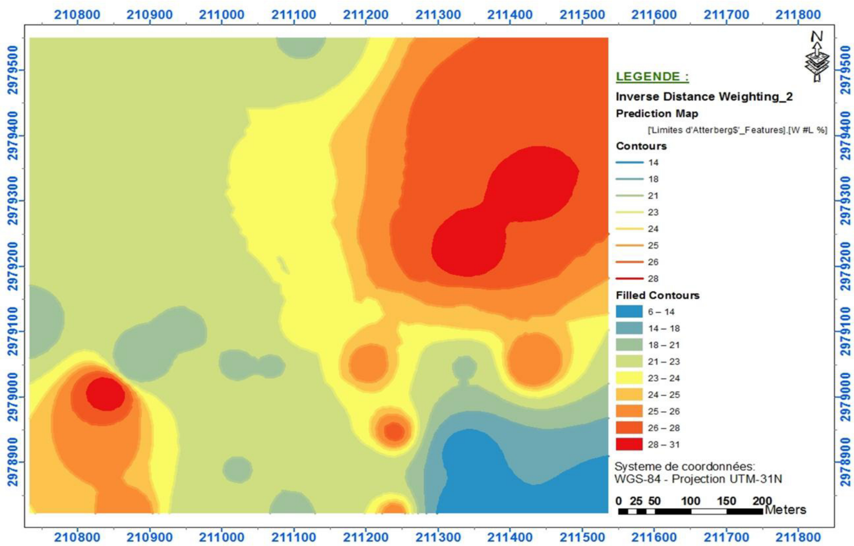

Fig. 8. Liquid limit model obtained on the basis of the IDW spatial interpolation.

- *With ordinary kriging (OK)*

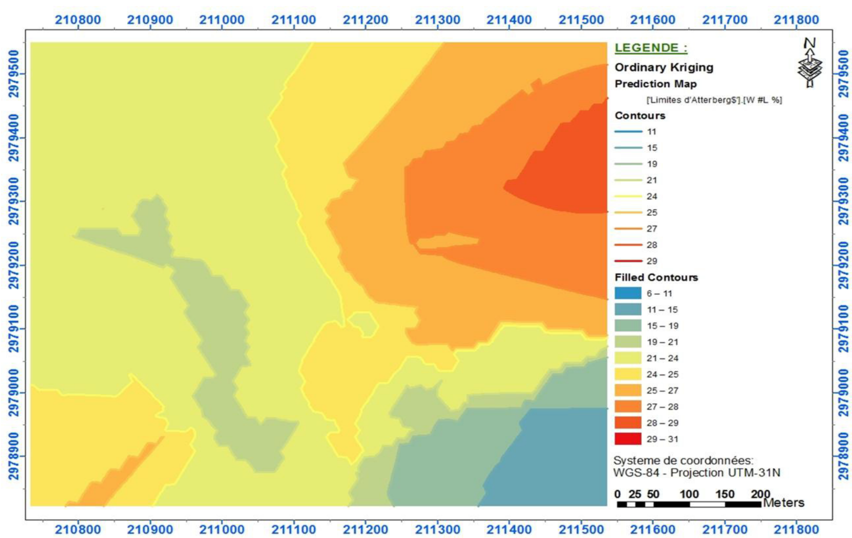

Fig. 9. Liquid limit model obtained by the ordinary kriging method (OK).

Discussion

In this work, two approaches to interpolation methods, one deterministic (IDW) and the other probabilistic (OK), were used.

The method known as "Inverse Distance Weighting" (IDW) calculates, for each point to be estimated, the average of the experimental values of its neighbors, favoring the closest points; the weighting factors are therefore calculated proportionally to the inverse of the distance. This method makes it possible to obtain grids very quickly, but creates circular zones around the observed values.

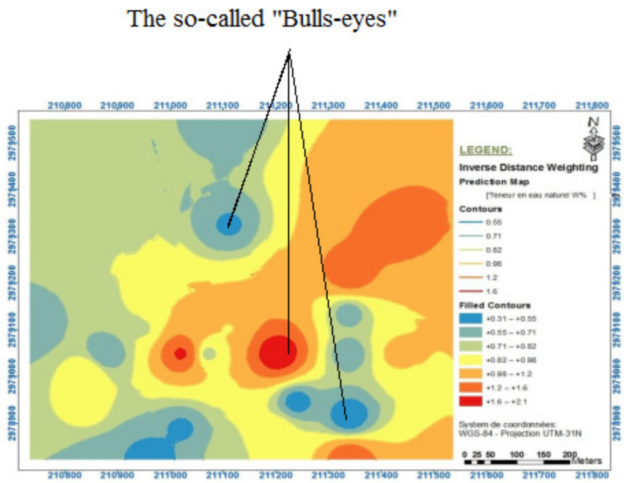

Fig. 10. An example drawback of the IDW interpolation model so called "Bulls eyes"

Unsightly artefacts are the so-called "Bulls-eyes" – these are circular areas of equal values around the known data points. However, applying a variation of the IDW-Interpolation developed by Shepard (1968) can reduce the Bulls-eyes.

This technique nevertheless has flaws: it ignores the spatial structure of the variable and thus produces very smooth interpolated surfaces, in addition no statistical criteria for judging the accuracy of the prediction are formulated. In IDW only known z values and distance weights are used to determine unknown areas.

Kriging is a statistical method that makes use of a variograms to calculate the spatial autocorrelation between points at graduated distances. It uses this calculation of spatial autocorrelation to determine the weights that should be applied at various distances. Spatial autocorrelation is determined by taking squared differences between points.

4. Conclusion

While deterministic methods, such as inverse distance weighting (IDW), use the Euclidean concept of distance to calculate the weights that will be applied to each sample data, the kriging weights are based not only on the distance between the measured points and the prediction location but also on the overall spatial arrangement among the measured points. To use the spatial arrangement in the weights, the spatial autocorrelation must be quantified. Thus, in Ordinary Kriging, the weight depends on a fitted model to the measured points, the distance to the prediction location, and the spatial relationships among the measured values around the prediction location. Variogram is an important input in kriging interpolation. It is a measure of spatial correlation between points.

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